



Oxford Cambridge and RSA

Thursday 6 June 2019 – Afternoon

A Level Further Mathematics A

Y541/01 Pure Core 2

Time allowed: 1 hour 30 minutes



You must have:

- Printed Answer Booklet
- Formulae A Level Further Mathematics A

You may use:

- a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question.
- The acceleration due to gravity is denoted by $g\text{ m s}^{-2}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION

- The total mark for this paper is **75**.
- The marks for each question are shown in brackets [].
- **You are reminded of the need for clear presentation in your answers.**
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **8** pages.

Answer **all** the questions.

1 In this question you must show detailed reasoning.

(a) By using partial fractions show that $\sum_{r=1}^n \frac{1}{r^2 + 3r + 2} = \frac{1}{2} - \frac{1}{n+2}$. [5]

(b) Hence determine the value of $\sum_{r=1}^{\infty} \frac{1}{r^2 + 3r + 2}$. [2]

2 (a) A plane Π has the equation $\mathbf{r} \cdot \begin{pmatrix} 3 \\ 6 \\ -2 \end{pmatrix} = 15$. C is the point $(4, -5, 1)$.

Find the shortest distance between Π and C . [3]

(b) Lines l_1 and l_2 have the following equations.

$$l_1: \mathbf{r} = \begin{pmatrix} 4 \\ 3 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix}$$

$$l_2: \mathbf{r} = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix} + \mu \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

Find, in exact form, the distance between l_1 and l_2 . [5]

3 In this question you must show detailed reasoning.

Show that $\int_5^{\infty} (x-1)^{-\frac{3}{2}} dx = 1$. [5]

4 A 2-D transformation T is a shear which leaves the y -axis invariant and which transforms the object point $(2, 1)$ to the image point $(2, 9)$. A is the matrix which represents the transformation T .

(a) Find A . [3]

(b) By considering the determinant of A , explain why the area of a shape is invariant under T . [2]

5 A particle of mass 2 kg moves along the x -axis. At time t seconds the velocity of the particle is $v\text{ ms}^{-1}$.

The particle is subject to two forces.

- One acts in the positive x -direction with magnitude $\frac{1}{2}t\text{ N}$.
- One acts in the negative x -direction with magnitude $v\text{ N}$.

(a) Show that the motion of the particle can be modelled by the differential equation

$$\frac{dv}{dt} + \frac{1}{2}v = \frac{1}{4}t. \quad [1]$$

The particle is at rest when $t = 0$.

(b) Find v in terms of t . [5]

(c) Find the velocity of the particle when $t = 2$. [1]

When $t = 2$ the force acting in the **positive** x -direction is replaced by a constant force of magnitude $\frac{1}{2}\text{ N}$ in the same direction.

(d) Refine the differential equation given in part (a) to model the motion for $t \geq 2$. [1]

(e) Use the refined model from part (d) to find an exact expression for v in terms of t for $t \geq 2$. [3]

- 6 A is a fixed point on a smooth horizontal surface. A particle P is initially held at A and released from rest.

It subsequently performs simple harmonic motion in a straight line on the surface. After its release it is next at rest after 0.2 seconds at point B whose displacement is 0.2 m from A . The point M is halfway between A and B .

The displacement of P from M at time t seconds after release is denoted by x m.

- (a) On the axes provided in the Printed Answer Booklet, sketch a graph of x against t for $0 \leq t \leq 0.4$. [4]
- (b) Find the displacement of P from M at 0.75 seconds after release. [2]

- 7 In an Argand diagram the points representing the numbers $2 + 3i$ and $1 - i$ are two adjacent vertices of a square, S .

- (a) Find the area of S . [3]
- (b) Find all the possible pairs of numbers represented by the other two vertices of S . [4]

8 **In this question you must show detailed reasoning.**

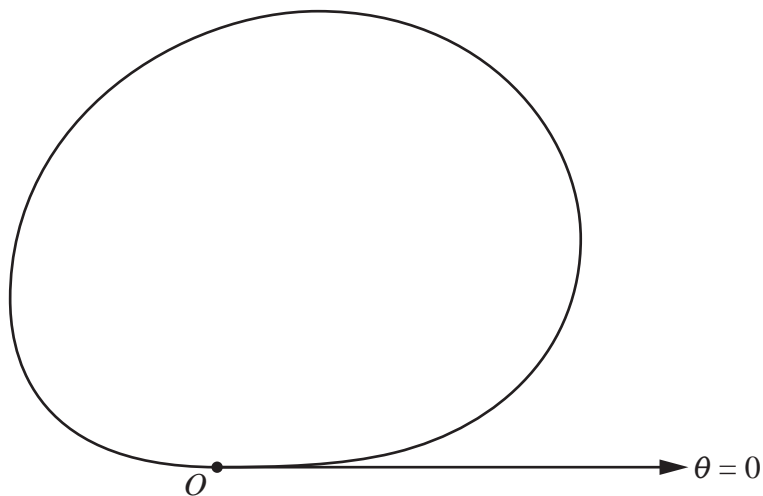
- (a) By writing $\sin \theta$ in terms of $e^{i\theta}$ and $e^{-i\theta}$ show that

$$\sin^6 \theta = \frac{1}{32}(10 - 15 \cos 2\theta + 6 \cos 4\theta - \cos 6\theta). \quad [5]$$

- (b) Hence show that $\sin \frac{1}{8}\pi = \frac{1}{2}\sqrt[6]{20 - 14\sqrt{2}}$. [3]

9 In this question you must show detailed reasoning.

The diagram below shows the curve $r = \sqrt{\sin\theta}e^{\frac{1}{3}\cos\theta}$ for $0 \leq \theta \leq \pi$.



(a) Find the exact area enclosed by the curve. [4]

(b) Show that the greatest value of r on the curve is $\sqrt{\frac{\sqrt{3}}{2}}e^{\frac{1}{6}}$. [7]

10 (a) Use differentiation to find the first two non-zero terms of the Maclaurin expansion of $\ln\left(\frac{1}{2} + \cos x\right)$. [4]

(b) By considering the root of the equation $\ln\left(\frac{1}{2} + \cos x\right) = 0$ deduce that $\pi \approx 3\sqrt{3 \ln\left(\frac{3}{2}\right)}$. [3]

END OF QUESTION PAPER

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